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Data Reconstruction Attack Against Principal Component Analysis

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We formalize a data reconstruction attack theory against Principal Component Analysis (PCA) by extending a former work about Membership Inference Attack (MIA) against PCA.

Aim of the Membership Inference Attack (MIA)

Given a trained ML model and some data point, decide whether this point was part of the model's training sample or not.

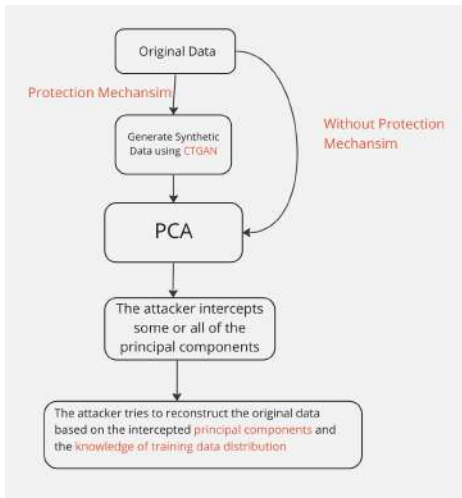
The goal of the adversary in a reconstruction attack is to extract the data used in the **training or inferences** of a machine learning model.

MIA against Principal Component Analysis(PCA)

- MIA against PCA [2] was studied for the first time
- The attacker intercepts some of the principal components and infers whether a particular sample participated in the computation of principal components.
- The theory is that the samples belonging to the training set will incur lower reconstruction error in comparison with the samples not belonging to the training set.

- Suppose X is an original data matrix of size $n \times p$ after subtracting the mean. Let V be the $p \times k$ matrix of some k eigenvectors to reduce the dimension.
- The matrix of PCA projection scores (Z) with the dimension $n \times k$ is $Z = XV$. To reconstruct all the original variables from a subset of principal components/eigenvectors, we can map it back to p dimensions with V^T .
- Reconstructed matrix, $\hat{X} = ZV^T$. Since we have a projection scores matrix, $Z = XV$, we obtain $\hat{X} = XVV^T$.
- We do not have access to the original data X ; we assume that the attacker has knowledge about the distribution of X . Therefore, the attacker can synthesize the data X_{syn} with a similar distribution as X and reconstruct the original data using $\hat{X} = ZV = X_{syn}V^T V$.

- We use a Conditional Tabular Generative Adversarial Network (CTGAN) to generate the synthetic data
- To show experimental results, we generate the synthetic data using different percentages of records from the original data, including {10%, 30%, 50%, 70%, 100%}



Data reconstruction attack against Principal Component Analysis

Description of datasets

Dataset	Number of Samples	Number of Attributes
Heart-scale	270	13
Mushrooms	8124	112
a9a	32561	123

- **No Protection Mechanism:** the data curator uses no protection mechanism at all
- **Differentially Private Principal Component Analysis (DPPCA):** the data curator applies DPPCA, which involves perturbing the covariance matrix

Definition

Suppose S is the synthetic data obtained after the alignment, and O is the original data. Let n be the total number of samples in the original and the synthetic data, O_j be the value of the sensitive attribute from the original data, which the attacker aims to infer, and S_j is the inferred attribute in the synthetic data corresponding to the sensitive attribute O_j . Let δ be the deviation between the original and the synthetic attribute that can be tolerated to measure the level of inference for a record. The lower the δ , the closer the values of S_j and O_j must be to each other. The Reconstruction Accuracy, *I.A.*, for the continuous attributes, is defined as follows:

$$R.A. = \frac{\#\left\{\hat{S}_j : \left|\frac{O_j - S_j}{S_j}\right| \leq \delta, j = 1 \dots n\right\}}{n} \quad (1)$$

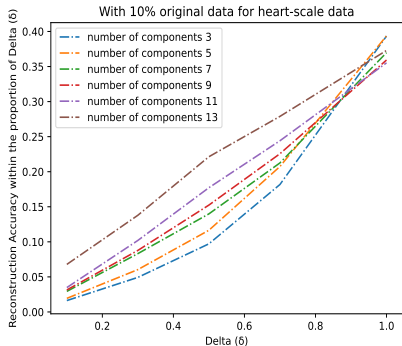
where $\#$ means count. I.A. is the percentage of inferred entries for which the relative errors are within δ .

For the categorical data, the above formula is more strict (as we are counting only the **exact matches**) and changes to

$$R.A. = \frac{\#\left\{\hat{S}_j : O_j == S_j, j = 1 \dots n\right\}}{n} \quad (2)$$

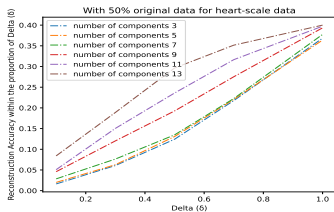
R.A. on the heart-scale data (CTGAN) I

- It is noted that there is **not much difference** in the R.A. when the CTGAN uses less percentage (e.g., 10%) of samples from the original data compared to using all the samples from the original data for generating the synthetic data.

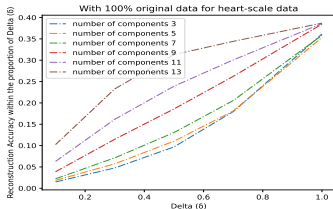


Used 10% of the original data

R.A. on the heart-scale data (CTGAN) II

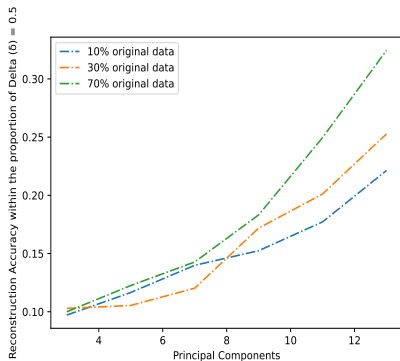


50% of the original data



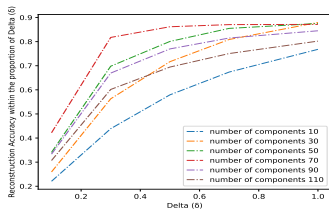
Used 100% of the original data

R.A. on the heart-scale data (CTGAN) III

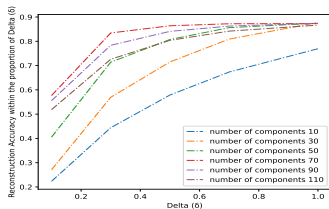


R.A. vs. no. of principal components with $\delta=0.5$

R.A. on the a9a data (CTGAN) I

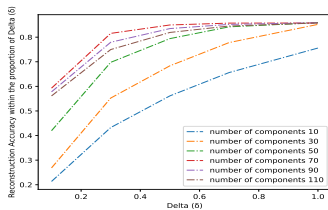


10% original data

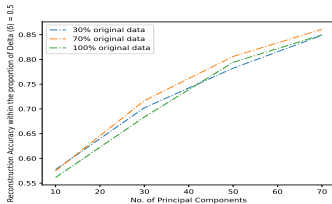


50% original data

R.A. on the a9a data (CTGAN) II

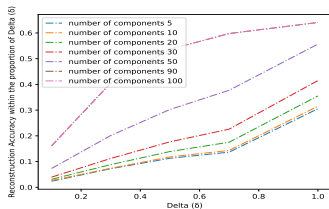


100% original data

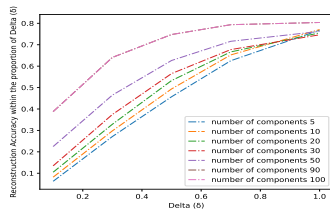


R.A. vs no. of principal components with $\delta = 0.5$

R.A. on the mushroom data(CTGAN) I

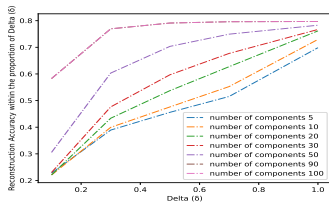


10% original data

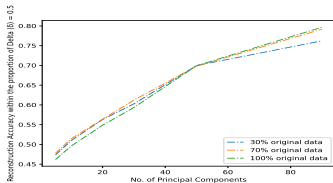


50% original data

R.A. on the mushroom data(CTGAN) II



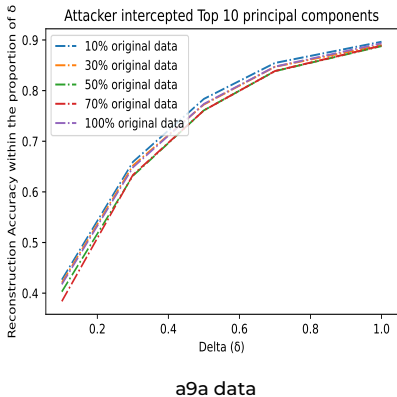
100% original data



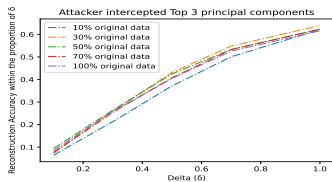
R.A. vs no. of principal components with $\delta = 0.5$

No Protection mechanism I

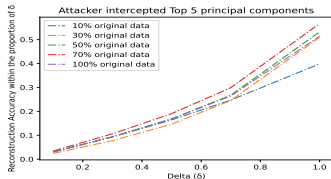
- When no protection mechanism is used, we show that the R.A. **increases** in comparison with the case when DPPCA is used, and when the principal components are computed on the synthetic dataset.



No Protection mechanism II



Heart-scale data



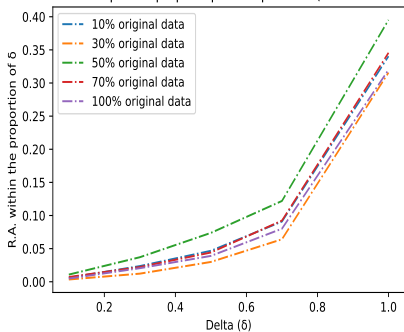
Mushrooms data

R.A. without protection mechanism prior to the computation of principal components

Results with DPPCA on the heart-scale data I

- Lesser the value of ϵ (**higher privacy**), the shallower the graph for reconstruction accuracy (**less reconstruction**).

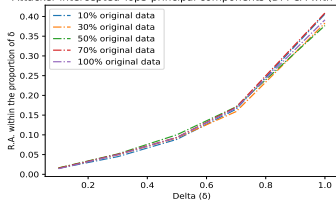
Attacker intercepted Top3 principal components (DPPCA with $\epsilon=0.01$)



$\epsilon = 0.01$ for DPPCA

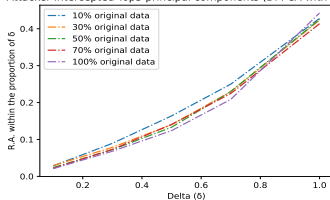
Results with DPPCA on the heart-scale data II

Attacker intercepted Top3 principal components (DPPCA with $\epsilon=0.1$)



$\epsilon = 0.1$ for DPPCA

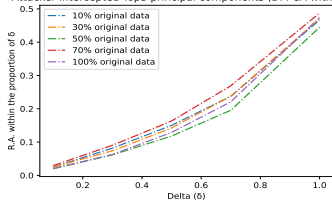
Attacker intercepted Top3 principal components (DPPCA with $\epsilon=0.5$)



$\epsilon = 0.5$ for DPPCA

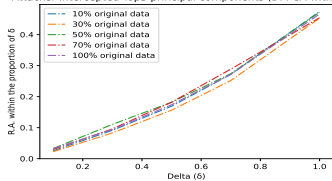
Results with DPPCA on the heart-scale data III

Attacker intercepted Top3 principal components (DPPCA with $\epsilon=1$)



$\epsilon = 1$ for DPPCA

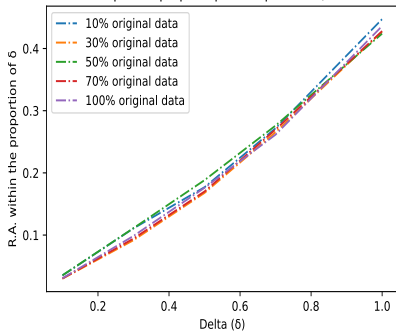
Attacker intercepted Top3 principal components (DPPCA with $\epsilon=2$)



$\epsilon = 2$ for DPPCA

Results with DPPCA on the heart-scale data IV

Attacker intercepted Top3 principal components (DPPCA with $\epsilon=5$)



$\epsilon = 5$ for DPPCA

- We demonstrated a **data reconstruction attack theory** against Principal Component Analysis.
- We compared two defense strategies, including **DPPCA**, and **synthetic data** against the proposed attack.

Thank You Very Much

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